The Craft's Use of Geometry in 16th c. Germany: A Means of Social Advancement? Albrecht Dürer & after

By
Sibylle Gluch, Ph.D (German Studies)
University of Dresden (Technische Universität Dresden

By the sixteenth century geometry had been important to the crafts for a long time. Yet the geometry of the medieval craftsman fundamentally differed from the geometry taught within the educational circle of the liberal arts. The latter was defined as the 'art of measurement.' It constituted as a theoretical art that focused on the teaching of Greek mathematicians such as Euclid and a practical one that dealt with the computation of lines, surfaces and bodies. From the High Middle Ages onwards both aspects interacted in many ways. The geometry of the crafts, on the other hand, involved neither mathematical rigour nor mathematical proofs. Equally, it did not use computations, but consisted solely of 'the construction and physical manipulation of simple geometrical forms' in order to 'solve technical problems of design and building' (Shelby, 1972, p. 409). Lon Shelby has demonstrated its simple and non-mathematical character by examining the booklets of two German craftsmen, the stonemason Matthäus Roriczer (c. 1440-95) and the goldsmith Hans Schmuttermayer (last mentioned 1518). Both wrote about the construction of gablets and pinnacles and neither of them gave the slightest indication that they knew of the existence of any kind of geometry other than their own. Thus, the constructive geometry of the craftsman was cut off from the theoretical and practical geometry of the educated world. It was the sixteenth century that saw the first attempt to cross this border when Albrecht Dürer published two geometrical books.

Albrecht Dürer: Mathematicizing Constructive Geometry

Early on in his career Dürer (1471-1528) planned to write a concise textbook on painting, which he provisionally titled *Ein Speis der Malerknaben* (*Nourishment of the Painter's Apprentice*, in the following referred to as *Malerbuch*). According to surviving notes on the table of contents, the book would have dealt with nearly every aspect of this art. Dürer intended to discuss the choice and education of the apprentice, the proportional canons of horses, humans, and buildings, the construction of perspective, the theory of colour, light, and shadow, as well as the question of what salary the painter should demand for his work. As this list is by no means exhaustive, the fact that the project remained unfinished might come as no surprise. However, the plan did not fail completely. Dürer published two treatises that clearly originated from the intended *Malerbuch,* called *Unterweisung der Messung* (*Instruction on Measurement*, 1525), and *Vier Bücher zu menschlicher Proportion* (*Four Books on Human Proportion*, 1528, in the following referred to as *Proportionslehre*). Compared with the original project these books are not only limited in scope of content, but focus above all on the geometrical aspects of Dürer's art.

Indeed, the *Unterweisung* provides a basic course in geometry, which Dürer organized in four books. The first starts at the very beginning and defines point, line, plane and body, however, its actual theme is lines. Dürer describes their general characteristics and forms as well as special cases such as parallels. He demonstrates the construction of a variety of lines and curves, among them parabola, hyperbola and ellipse, and explains further geometrical problems such as bisecting a given line, or finding the centre of a circular arc. The second book deals with planes. First, Dürer clarifies the different forms of angles (acute, right, obtuse), triangles (equilateral, isosceles, scalene) and quadrangles (square, rectangle, rhombus, trapezium). He then constructs a number of different polygons with compass and straightedge, some of which are mathematically strictly correct, while others attain only approximate results. Most importantly, Dürer was the first to differentiate the mathematical quality of the former and latter by designating them as 'mechanical' (*mechanice* = approximate) or 'demonstrative' (*demonstrative* = accurate). Furthermore, he discusses area problems such as trebling the area of a given rectangle while maintaining its proportions, and some of the classical issues of geometry, namely the problem of squaring the circle and Pythagoras' theorem.

The caption of the third book reads 'von den Corperlichen dingen' (of the things corporeal), nonetheless, a systematic outline of geometrical bodies is offered in book four. The third, in contrast, calls the geometry of craftsmen to mind. For here Dürer teaches the construction of several architectonic forms such as columns including bases and capitals, and memorials, for example, for the Peasants' War. On this occasion, he refers to earlier parts of his teaching by recommending the spiral line for decorating the columns or using one of the curves of book one for constructing the tower roof. Thereby, the differences between Dürer's and Roriczer's and Schmuttermayer's teaching become obvious. The latter two provided recipes for the construction of a particular architectural form. Their instructions had to be followed step by step. Dürer, on the other hand, explains the basic geometric forms, which can then be combined in endless variations. Hence, his *Unterweisung* surpasses Roriczer's and Schmuttermayer's booklets in scope of content as well as in intellectual quality. Yet, the third book has even more to offer. In its second half it deals with a series of quite varied tasks, for example, how to measure the height of a tower with the help of a right-angled triangle and how to make a sundial. It also exemplifies how letterings on tall buildings should be gradually enlarged in order to compensate for the occurring optical distortions and gives detailed instructions concerning the geometrical construction of Roman and Gothic letters.

Book four, finally, seems more geometrical in comparison with the third. It describes the construction of regular and irregular solids, which are illustrated with templates that the reader could cut out and fold in order to obtain the actual body. In addition, Dürer includes an old problem of geometry: the doubling of (the volume of) the cube. Here he not only gives three different solutions which are ascribed to the ancient mathematicians or philosophers Sporus, Plato, and Heron, but he also proves the correctness of Heron's method. This is the only proof to be found in Dürer's *Unterweisung*, and it is not his own. Rather, he copied it together with Sporus's, Plato's and Heron's answers from Johannes de Muris' (c. 1290-1351/61) treatise *De arte mensurandi* (Clagett, III, p. 1168). The last part of book four, however, is built on Dürer's own studies. Here, as one of the first in the German vernacular to do so, Dürer describes the theory and practice of central perspective and thereby closes his treatise with the explanation of a technique that was about to conquer the world of pictorial representation. Given this account of Dürer's *Unterweisung* the booklets of the fifteenth century craftsmen Roriczer and Schmuttermayer fall behind significantly. First, they are restricted in content, but more importantly, they differ in their treatment of geometry.

For Roriczer and Schmuttermayer geometry was a constructional tool that allowed the design and transformation of geometrical figures, while guaranteeing the preservation of proportionality at the same time. For these authors, geometry was the 'art of construction,' which was equally distant from the geometry of Euclid and that of practical geometry as taught in monasteries, Latin schools and universities. Hence, Roriczer's and Schmuttermayer's references to geometry as "hohe und freie Kunst" (high and free art) only outwardly touch on the traditional classification of geometry as one of the liberal arts. Here, both authors use a phrase that shows no connection to the concept of the liberal arts or *artes liberales* as established in the Latin-speaking world.

Dürer, on the other hand, clearly followed the model of practical geometry (Peiffer, p. 91). He defined geometry as the 'art of measurement' and arranged his *Unterweisung* according to its common subdivisions: book one deals with lines, book two with planes, and book four with bodies. However, Dürer's methods as such do not comply with those of practical geometry. The latter was commonly interested in computational problems, which it usually solved with the help of arithmetic. Dürer, by contrast, propounds constructional problems, which he solves by means of constructive geometry. Yet, compared to the booklets of Roriczer and Schmuttermayer, the geometry used in the *Unterweisung* appears transformed because Dürer links the geometry of the craftsmen to the geometry of the universities. Thus, he not only presents a number of new constructions, but also marks the difference between accurate and approximate solutions, which at least within workshop practice had been previously unknown. Even if Dürer did not contribute to theoretical geometry himself, he introduced it to the craftsman. In his *Unterweisung* he repeatedly refers to Euclid, and he incorporated the first mathematical proof in a German geometric book even though he copied it. In this way he created a link between the world of the craftsman and the world of the scholar (Olschki, I, pp. 419-23).

As a result, geometry in Dürer's treatises (*Unterweisung* and *Proportionslehre*) did not only serve as a technical tool, but performed a social function too: it had the power to elevate the craft that used it—in Dürer's case painting—to the rank of a science. In other words, it transformed an *ars mechanica* into an *ars liberalis*. We will examine this process more closely. First, however, it is necessary to take a glance at the Italian Renaissance, for it was an Italian humanist, amateur painter, and architect who initially stressed the importance of geometry for Dürer's art.

Leon Battista Alberti (1404-72) held that without geometry painting cannot reach perfection. Indeed, he declared that without knowledge of geometry painting cannot be mastered at all, as its very methods could not be understood (*De pictura*, III.53). Consequently, the ignorant painter could not hope to achieve even a mediocre standard in his art. In accordance with humanist tradition Alberti referred to classical antiquity and presented the painter Pamphilus as the old and noble father of his thought (De picture, III.53). As Pliny records, Pamphilus, teacher of the famous Apelles, was 'the first painter who was skilled in all sciences, but particularly in arithmetic and geometry without the aid of which, he announced, pictorial art could not attain perfection' (Plinius / König, p. 64). It was through the influence of this learned man that in the whole of Greece all boys of free birth were taught how to draw. In the end, graphic art acquired the first place among the liberal arts. The erudite Pamphilus thus achieved a general promotion of the status of his art, but this did not survive the course of time. Medieval society regarded painting as one of the crafts, i.e. as one of the *artes mechanicae*. Alberti hoped to correct this assessment and to revalue painting's rank.

Under his aegis, painting and geometry entered into a symbiotic relationship, which he proclaimed in two of his treatises: *De pictura* (Italian and Latin c. 1435/36) and *Elementae picturae* (Italian c. 1435/36, Latin 1450-55). Particularly the latter illustrates this correlation most clearly. The *Elements of Painting* do not give—as one might expect—explanations regarding colour or light, but provide definitions which 'stem from the mathematicians' (*Elementae picturae*, § A). Yet Alberti did not simply confront the painter with geometry. Rather he embarked on the task of making the one art completely compatible with the other. For this reason, he added his own explanations to the former definitions, which he also modified so that they might better serve the painter's needs. For example, Alberti defines the point in a mathematical sense as 'the thing that cannot be divided,' and in a pictorial sense as 'a dot so small that no hand can make a smaller one' (*Elementae picturae*, §§ A and C). In a similar way he explains the line, which mathematically speaking is 'length without width,' but graphically it is a 'very fine stroke running from one point to the other' (Elemetae picturae, §§ A and C). In this manner Alberti acknowledged both painting's particular realization of the abstract elements of geometry and the ultimate kinship between the two arts: painting did not merely use geometric constructions but perceived the world in the geometric terms of lines, surfaces, bodies, and their proportions.

Alberti's focus on geometry was closely connected with the Renaissance invention of linear perspective, which incidentally he was the first to describe (*De pictura*, I.19-21). With the introduction of this technique the painter faced a new challenge: to produce a correct image of the perceived world. The term 'correct' here carries a mathematical or geometrical meaning, since Alberti defined the painting as 'cut through the visual pyramid' (*De pictura*, I.12). Not only could this 'cut' be represented by means of geometry (e.g. geometrical optics), it could also be geometrically constructed. Hence, in order to perform his art properly the painter needed to master geometry. However, Alberti did not ask for the kind of geometry that Roriczer and Schmuttermayer used. On the contrary, he profoundly criticized such a mechanical approach, while commenting on common perspective methods of his time (*De pictura*, I.19). Because it neglected the standpoint of the observer, workshop practice yielded only approximate results. Yet, Alberti's aim was to reproduce faithfully what one saw. Hereby, geometry became a means to study nature (Westfall, pp. 495-98).

With the help of geometry the painter could investigate the natural world. As a geometer he studied the lines, surfaces, bodies, and proportions of the world around him in order to transfer his findings into his work. The painting thus achieved represented a true image of the world and, furthermore, conveyed knowledge of its structure. In turn, the painter who had accomplished such a work could not be called a 'craftsman' anymore. Hence, by relying on the right kind of geometry Alberti had opened the door to the *artes liberales* for painting.

It is well known that Dürer had a keen interest in advancing his own social status and repeatedly presented himself as a noble and learned man. So the fact that he shared Alberti's understanding of geometry does not come as a big surprise. In addition to the inspiration offered by Alberti's work, however, Dürer referred to an old Christian tradition in order to emphasize the relationship between nature and geometry. On the grounds of Book of Wisdom 11.21—'Thou creator of the world hast ordered everything according to measure, number, and weight.'—the church fathers already believed that the wisdom of God is expressed in the order of numbers. Furthermore, they held that this order and thereby the creation itself could be comprehend by man. As geometry is 'the art of measurement' could there be an instrument more suited to discern the divine order of the world? In an unpublished note written in 1512/13 and intended to introduce the *Malerbuch* Dürer declares that he wants to ground his project on measure, number, and weight (Rupprich, II, p. 104). In another comment dated 1512 he points out the consequences of such new interrelation between painting and geometry: 'The measurements of the earth, the water, and the stars have become comprehensible because of the painted image, and still many more things are to come to the understanding of humankind through indication of the painted image' (Rupprich, II, p. 113). Consequently, painting had become an instrument for the study of nature. In its products, the pictures, it revealed the order of the world.

But painting could only do so by relying on the right kind of geometry. Hence, Dürer's central purpose was to teach his colleagues the 'art of measurement.' For this reason he published the *Unterweisung*, which is described above, and the *Proportionslehre* about which a few words may be necessary here. On first glance, a book on human proportion does not have much to do with geometry. Yet 'proportions' are the very domain of geometry, and Dürer constructed the human body just as any other geometrical shape. In the *Proportionslehre*, he teaches two methods of measuring it and also explains how to proportionally enlarge or reduce its size in length, width and heights. Moreover, at the end of the fourth book he substitutes cubes and rectangles for the human body in order to study and construct it in movement. Most obvious here, but true of the whole treatise, geometry serves as a model and means to study nature, and again the painter will present his findings in his work.

It goes without saying that neither Alberti's nor Dürer's ideas on art only revolved around geometry, but the latter did play an important part in their thinking. As one of the mathematical sciences geometry provided firm ground for painting's techniques and results. Dürer said that ideally the painter could prove the truthfulness and accuracy of his work with it (*Proportionslehre*, fol. T3v). Furthermore, it was one way to distinguish painting from the rest of the crafts and one avenue for claiming the nobility of the painter: the liberal art of geometry elevated his mechanical art. Thus, Alberti and Dürer demonstrated a strategy that principally could be employed for a number of crafts. Indeed, it seems that the hint was taken, for after Dürer's treatises quite a number of German texts on the subject appeared. Whether their authors followed Dürer's intention I will examine now.

Scientific Instrument or Mechanical Tool: Geometry after Dürer

Dürer's undertaking to publish textbooks for the teaching of his art proved very successful in one particular respect: it prompted others to emulate him. Consequently, the sixteenth century saw the appearance of a number of books dealing with pure and applied geometry in the German vernacular. The titles considered here have been chosen for the following reasons: First, all of these books deal with perspective and / or proportion and thus with topics that Dürer had introduced to German literature. Second, the authors either address craftsmen or were craftsmen themselves. Third, they verbally explain their constructions, which therefore can be assessed. Lacking this last element, the numerous pattern books that were published during the sixteenth century are excluded here. In chronological order the remaining titles are:

- Johann II of Simmern, *Eyn schön nützlich büchlin vnd vnderweisung der kunst des Messens / mit dem Zirkel / Richtscheidt oder Linial* (*A Beautifully Useful Booklet and Instruction on the Art of Measurement with Compass, Straightedge orRruler*), Simmern 1531;
- Erhard Schön, *Unnderweissung der proportzion vnnd stellung der possen* (*Instruction on the Proportion and Positioning of Figurines*), Nuremberg 1538;
- Augustin Hirschvogel, *Ein aigentliche vnd grundtliche anweysung / in die Geometria* (*A True and Thorough Instruction into Geometry*), Nuremberg 1543;
- Sebald Beham, *Kunst vnd Lere Büchlin* (*Art and Training Booklet*), Frankfurt a. M. 1552;
- Heinrich Lautensack, *Des Circkels vnnd Richtscheyts / auch der Perspectiua / vnd Proportion der Menschen vnd Rosse / kurtze / doch gründtliche vnderweisung / des rechten gebrauchs* (*A Brief but Thorough Instruction on the Right Use of compass and Straightedge, also Perspective, and the Proportions of Humans and Horses*), Frankfurt a. M. 1564;

- Hans Lencker, *Perspectiva* (*Perspective*), Nuremberg 1571.

With the exception of the Palatine Johann II of Simmern all authors practiced a craft. Erhard Schön and Sebald Beham trained as painters, Heinrich Lautensack and Hans Lencker as goldsmiths, the versatile Augustin Hirschvogel came from a family of glass painters. Apart from Lencker all of these authors address craftsmen in either title or introduction to whom they particularly offer their works. Finally, most of them refer to Dürer, and characterizing his writings as too difficult they accordingly seek to present more easily comprehensible instructions. Whether this ambition affected their treatment of geometry we will see shortly, but first I shall say a few more words about the books.

As should be expected, the books differ in content and style. Johann dedicates his treatise to perspective, which he wishes to describe in a manner easy to understand. Many illustrations guide the learner from very simple constructions to more complicated ones. The treatise closes with a brief description of the proportion of man. The latter is the central topic in Erhard Schön's little booklet, again written for the young and inexperienced. In addition, Schön includes notes concerning the perspective foreshortening of figures in a room, equine proportions, and the construction of a helmet. Hirschvogel's *Geometria* consists of three parts. The first deals with lines and planes, the second with the construction of the Platonic solids, and the third with perspective. Unlike Johann, who demonstrates the construction of whole landscapes and interior scenes, Hirschvogel restricts his instructions to geometric bodies. Sebald Beham, on the other hand, completely ignores perspective. His booklet describes a few geometrical constructions and the construction of human heads in different views. It closes with illustrations that serve as a pattern-book. Lautensack seems to have followed Dürer's example very closely. His treatise summarizes *Unterweisung* and *Proportionslehre* in one book. First, it explains some basic geometric terms as well as the construction of a number of lines and planes. The second part deals with perspective, the third with human proportions. Moreover, Lautensack briefly touches on equine proportions. Such variety was not Hans Lencker's cup of tea. He specialized in perspective and his book shows all to well how advanced his knowledge was. With convoluted explanations and few illustrations it is not easy to follow, and certainly not suitable to the beginner whom all the other authors address. Despite such diversity, the books can all be called geometrical as their authors understood perspective and proportion in a purely geometrical way. Their central question was one of construction, whether it concerned objects in three dimensions or the bodies of humans and animals.

So how do the authors define geometry? Surprisingly they hardly mention the word. Heinrich Lautensack and Sebald Beham, for example, do not employ it once. Rather, Beham speaks of "Maß vnd außteylung des Circkels" (measure and distribution of the compass) and thus uses a phrase that is typical of the crafts. Similarly, Lautensack mentions 'Des Circkels vnnd Richtscheyts […] rechten gebrauchs' (the right use of compass and straightedge) in his title, but later only names perspective when talking about the varied content of his book. As a matter of fact, this latter art must have held a particular attraction, since Johann and Hirschvogel both declare that perspective is the 'art of measurement' (Johann, fol. A2r, Hirschvogel, fol. A2r)! While these remarks certainly show the emancipation of this newly discovered science that now occupies geometry's place, they also reveal the semantic inconsistency with which sixteenth century writers applied these terms and definitions.

Augustin Hirschvogel, for example, explicitly titles his book *A True and Thorough Instruction into Geometry*, but then gives its traditional definition to one of its subdivisions: perspective. The separate part of his illustrations, on the other hand, bears a striking caption that seems to refer to geometry as one of the liberal arts: "DAS BVCH GEOMETRIA IST MEIN NAMEN ALL FREYE KVNST AVS MIR ZVM ERSTEN KAMEN ICH BRING ARCHITEKTURA VND PERSPECTIVA ZVSAMEN" (The Book Geometry is my name / All free arts from me first sprang / I bring together architecture and perspective). Now, Hirschvogel seems to consider both architecture and perspective as geometrical arts, and furthermore, to credit geometry with being the 'first of all sciences.' But what does this brief remark actually reveal about Hirschvogel's understanding of geometry? Is it more than an embellishment? We have to be suspicious, for Hirschvogel employs the word 'geometry' in yet another way. Throughout his book he constantly uses the word in phrases like this: 'This geometry in the circle / draw like this' (fol. Cv), and 'To divide up this six-sided corpus into the geometry / do as follows' (fol. C2v). What does he mean here?

In a context like this, 'Geometria' stands for a particular kind of drawing: it basically serves as a synonym for ground plan. Such denotative restriction also occurs in the writings of Hans Lencker, who explicitly states: 'And so I will have meant and understood the word Geometria / as every thing's ground, which it would cover (if it were corporeal) in a perpendicular manner from above' (Lencker, 1567, fol. A2r). This specialized meaning may be a development of the sixteenth century, but we can find earlier traces. Matthäus Roriczer, for example, attempted to explain 'how and with what measurements the drawn-out stonework should come out of the ground of geometry' (Roriczer / Shelby, p. 82). Such statement can be read in a twofold way. The German word 'Grund' (ground) can mean base or foundation, in which case Roriczer emphasizes the correctness of his teaching. However, the word could also be understood as ground plan because this plan supplied the mason with all the information that he needed in order to construct the elevation, i.e. to draw the stonework out of the ground. Given the fact that Roriczer frequently uses 'Grund' when referring to the ground plan, he might well have had the second meaning in mind (cf. Roriczer / Shelby, pp. 88 and 90). Be that as it may, this particular understanding of the word 'geometry' was genuine to the crafts. It referred to the processes of constructing and drawing which both were part of, for instance, the mason's, the goldsmith's, and the painter's work.

Coming back to Hirschvogel's *Geometria* we can now see that the text contains semantic signs that point in two directions: the geometry of the liberal arts and the geometry of the crafts. As this is true for most of the texts in question, it is necessary to examine the geometric constructions. Their characteristics and quality will ultimately reveal which kind of geometry the authors employed. In order to ensure easy compatibility I will focus on the techniques of perspective and proportion, which—to come straight to the point—will show that none of the authors attached such significance to geometry as Dürer did.

Dürer presented a number of exact and approximate perspective techniques, which are nonetheless all similar in one respect; they clearly show the interconnections between original, observer, and image. For instance, he demonstrated a geometrical procedure which he called the 'nähere Weg' (closer way) since it was the abbreviated version of a more elaborate one. The task is to construct a foreshortened square, for example, one square of the chequerboard floor that is so notorious in fifteenth and sixteenth century paintings. The difficult bit is to find out where the far edge of that square would be. So Dürer first draws the ground line of his square. He adds the central vanishing point, a projection of the observer, and connects it with both ends of the ground line. Then he positions the picture plane and places the observer at whatever distance he likes. Now he traces the visual rays from the observer to both ends of the square. (Here, text and illustrations disagree. The text describes the correct procedure, while the illustration is erroneous.) The point of intersection of that longer ray and the picture plane determines where the rear end of the square has to be drawn.

Even without any knowledge of perspective it is obvious that this construction is based on the actual process of vision. It fixes the image as the observer saw it at a given time, from a given point of view. Therefore, the geometrical construction is the intermediary between nature and pictorial representation.

This is not the case with the methods that, for example, Johann of Simmern and Heinrich Lautensack recommended. Johann constructs his chequerboard floor by first drawing 'a triangle,' and then tracing the orthogonals, i.e. the lines that run into depth. Observer and picture plane are not mentioned and, moreover, the final construction is incorrect as Johann establishes the position of the horizontal lines by drawing two diagonals where only one would have been correct. Lautensack, on the other hand, constructs a single square similar to Dürer. However, he establishes near and far edge of that square by simply drawing two parallels at whatever distance he wishes. Then he places the central vanishing point and connects it with the two lower edges of his square: Voilà! The result is not necessarily wrong—but it does depend entirely on where observer and picture plane are thought to be. Also, Lautensack performs the second step before the first. Thus, the link between the process of vision and the perspective image is of a totally accidental nature. This assessment also applies to Hirschvogel who places geometric solids on a foreshortened chequerboard square. He ultimately constructs the solids correctly, but first he draws the far end of his square wherever it seems fit. He thus ignores the dependence of the image on the standpoint and position of observer and picture plane.

The only author besides Dürer to pay explicit attention to this fact is Lencker. He uses Dürer's method of the 'closer way' and clearly demonstrates that the rays of vision run from the observer to the observed. The far end of his square is drawn accurately at the point where the picture plane is cut. Hence, Lencker's construction abides the interconnection between geometry and nature that for Dürer was all-important. Yet, Lencker is only one out of four! We may conclude that Dürer's understanding of geometry was not wide-spread among craftsmen and will look at the respective techniques of proportion for confirmation.

Dürer had studied the topic of human proportion for many years. He sought to discover nature's proportional laws. He published the results of his efforts in his *Four Books on Human Proportion*, which by means of different body types give a systematized presentation of the data won before. In this manner, Dürer once more used the 'art of measurement' as an instrument for the study of nature—but this can hardly be said of his successors.

Four of the authors attempt to tackle the problem of human proportion, Johann of Simmern, Erhard Schön, Sebald Beham and Heinrich Lautensack. Johann criticizes the lack of proportional knowledge on the side of the painters, yet gives only very general advice. According to him male figures shall have broad shoulders, but slender hips, while female figures have to show an equal width throughout. As a rule, the hand shall not be wider than half of the face 'from the nose to the ear' (Johann, fol. G5v), while the elbow has to reach the hip. He continues in this manner, but the given examples should sufficiently illustrate that in his instructions neither the direct study of nature nor the thorough application of the 'art of measurement' play any part. Erhard Schön's teaching falls within the same category. He never mentions nature, to say nothing of taking it as a starting point. Rather, he recommends a pre-composed constructional system that consists of a mixture of geometrical forms and predetermined measurements. In this manner, Schön attains a natural-looking figure, but in a purely mechanical way. Understanding nature's proportional canons is of no interest to him, easy practicality is.

The same can be said of Beham's construction of faces. He took the idea from Dürer, whom he probably knew personally, but he oversimplifies it. In the first book of his *Four Books on Human Proportion* Dürer uses rectangular grids in order to create different views of the human head. However, he not only determines the inner structure of his grids according to the measurements of the human head, but, moreover, uses projective geometry in order to construct its profile view and even its ground plan. Beham, on the other hand, uses a grid with only nine equal squares, which in no way resemble the different lengths that a human face contains. Hereby, the grid is not a construction instrument, but merely a drawing aid, particularly as Beham does not even refer to measurement in the first place. Schön and Lautensack basically employ the grid in the way. The only difference is the number of squares that each grid contains. Lautensack, for instance, uses sixteen rather than nine. As a result his drawing might be somewhat finer. Nevertheless, all three authors reduce geometry or the 'art of measurement' to the status of a drawing tool.

An apparent exception to the rule is Lautensack's construction of the human figure, in which he follows Dürer's example. The illustrations already reveal Dürer's influence but, what is more, Lautensack also teaches a measuring method that Dürer had demonstrated before. First described by Alberti, this technique makes use of a measuring staff. The staff's height corresponds with that of the figure to be measured. Alberti divided its lengths into six equal parts called *pedes* (feet), each of which he then subdivided into ten equal parts called *unceolas* (inches) and these again into ten parts called *minuta* (minutes) (*De statua*, 7). Dürer refined Alberti's instrument by introducing yet another subdivision. Lautensack, on the other hand, simplified the procedure by using a coarser scale. Nevertheless, he actually measures the human figure, which Johann and Schön do not. Furthermore, he not only explains the proportion of a man, but also those of a woman and child.

All in all, his treatise is much more demanding than those of Johann, Beham and Schön, but easier to understand than Dürer's. In point of fact, it comprises Dürer's two books (*Unterweisung* and *Proportionslehre*) in one. Such abridgement had obvious advantages as Dürer's explanations are often quite tedious. Nonetheless, it held dangers too. For instance, while Dürer presented general types of the human body, Lautensack displayed only one so as to explain the technical procedure involved. Thus, the 'art of measurement,' which was the pivotal point in Dürer's writings, ultimately became a means to a practical end. It guided and regulated the process of drawing, but did not reveal the hidden structure of the world.

Conclusion

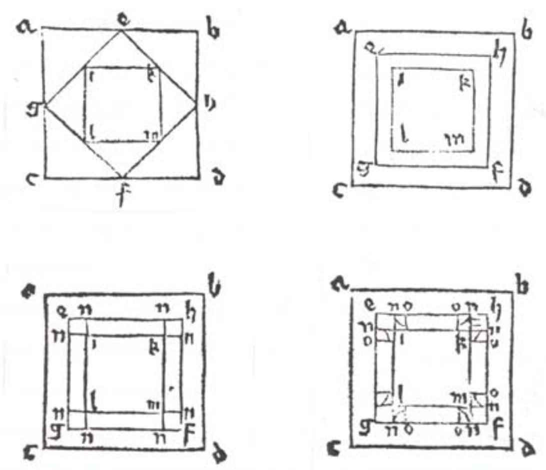
Albrecht Dürer first attempted to combine the geometry of the educated world with the one that was practiced by crafts. In his *Unterweisung der Messung*, he demonstrated the constructive geometry of the crafts, but at the same time he structured his book according to the common divisions of practical geometry that dealt with lines, planes, and bodies. Furthermore, he also hinted at theoretical geometry by repeatedly referring to Euclid's work. In Dürer's writings it is obvious that the role of geometry had changed. Whereas to Roriczer and Schmuttermayer it had been a design device, to Dürer it became an instrument of cognition: geometry revealed the order of the world. While the notion as such had a long tradition, Dürer's use of it originated in the Italian Renaissance. As did Alberti, he concluded that painting by employing geometry could impart knowledge about the structure of the natural world. As a consequence, painting could not be regarded as a craft or mechanical art anymore. Via the route of geometry it had entered the circle of the liberal arts.

Dürer had shown a way that could be taken by various crafts. Nonetheless later German authors did not emulate his example. On the contrary, the majority of the writers examined here continued to use geometry in the same mechanical way that Roriczer and Schmuttermayer had. To be sure, the constructions differ qualitatively, but altogether geometry remained a practical tool. In contrast to Dürer, his successors did not seek to link their respective crafts to the liberal arts. Indeed, their texts do not reveal whether any of them had a clear understanding of what the liberal arts are! At any rate, the writers show little knowledge of the liberal art of geometry—perhaps with the exception of Hirschvogel and Lencker.

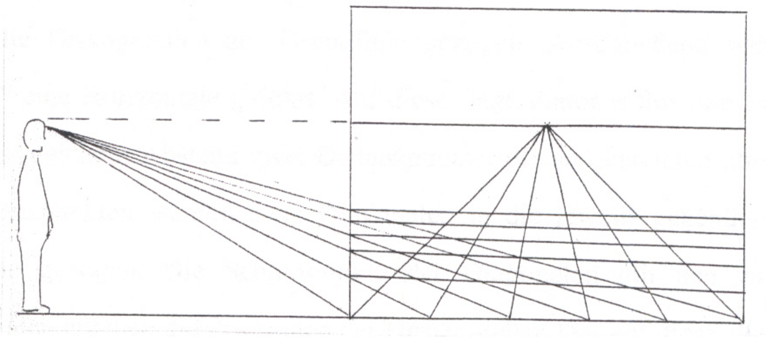
Hirschvogel certainly had a wider knowledge of geometry than his treatise reveals. He seems to have read and understood Dürer's *Unterweisung* and was himself an expert in cartography, which brought him the honour of being called 'mathematicus' (Schwarz, pp. 16-22). And yet, in his book he appears to be interested in the technical side of his constructions only, and neither Euclid nor any other 'old mathematician' appears. So why did he restrict his teaching in such a way? An explanation is offered by the author himself. Hirschvogel sought to explain the art of perspective for 'the use of the hand' ("handtbrauch", Hirschvogel, fol. A2r), i.e. for practical application. His purpose was to provide his reader with particular practical skills in order to protect the 'honest and useful arts' and their respective artists from decline and disregard (Hirschvogel, fol. A2r). Dürer, on the other hand, pursued a different aim, for he sought to advance one particular art, i.e. painting. His goal was not practicality, but perfection! As the dedications of his treatises display Classical antiquity and Renaissance Italy provide the background against which he measured German art. Thus, Dürer was far more ambitious than Hirschvogel, because he ventured to alter the foundation of his art, and to thereby distinguish it.

Neither Hirschvogel nor any other of our authors followed Dürer here. Rather they sought to impart technical knowledge, clearly defined in scope and ready to be applied in practice—except Hans Lencker. This goldsmith not so much wished to instruct his reader as to parade his own expertise and capability. His convoluted instructions and complicated perspective apparatus hardly comply with the ordinary craftsman's needs. Nor were they supposed to enhance the reputation of a particular craft—Lencker does not mention any. Hence, though his geometry resembles Dürer's in quality, in his treatise it still plays a different role, serving his personal distinction. In this he was successful; Lencker was offered a post at the Dresden court because of his writings (Kappel, p. 176).

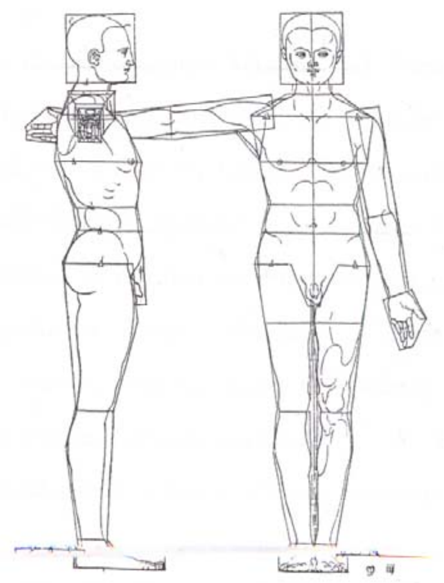
<u>Illustrations</u>



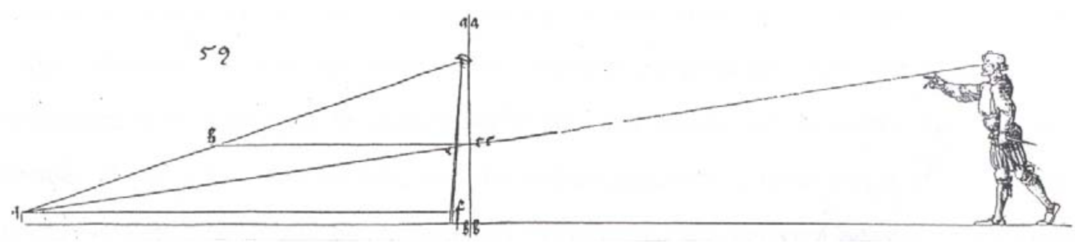
1. Matthäus Roriczer: Ground plane for a pinnacle



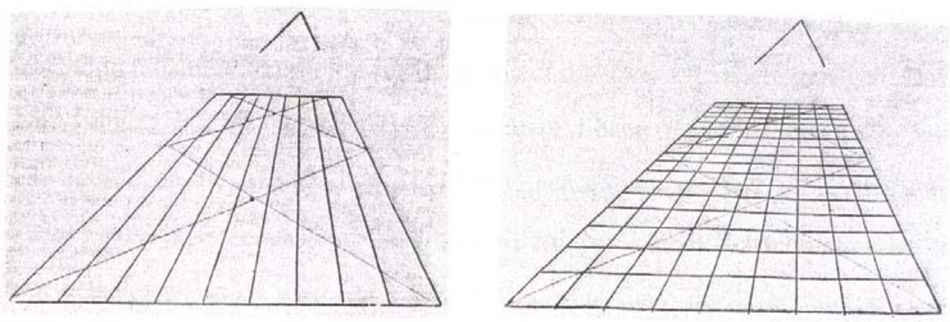
2. Leon Battista Alberti’s perspective construction



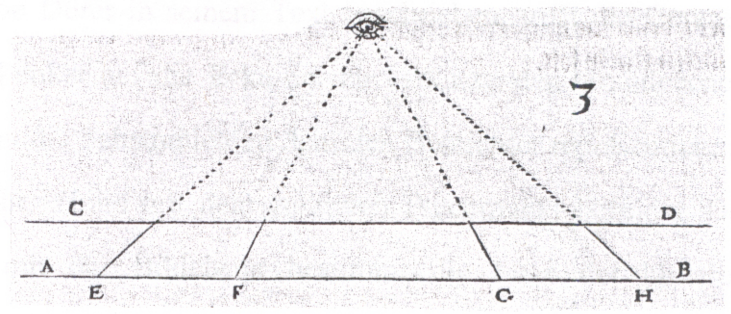
3. Albrecht Dürer: Stereometric figure (*Proportionslehre*, fol. Y3r)



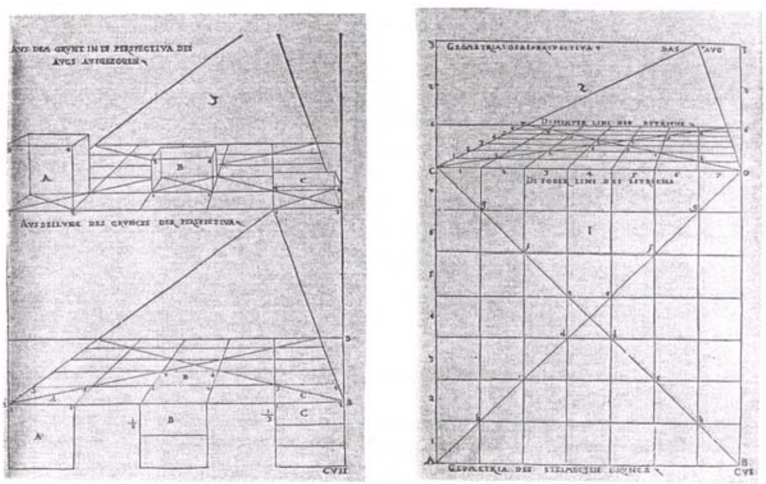
4. Albrecht Dürer: The ‚closer way’ (*Unterweisung*, fol. P4v)



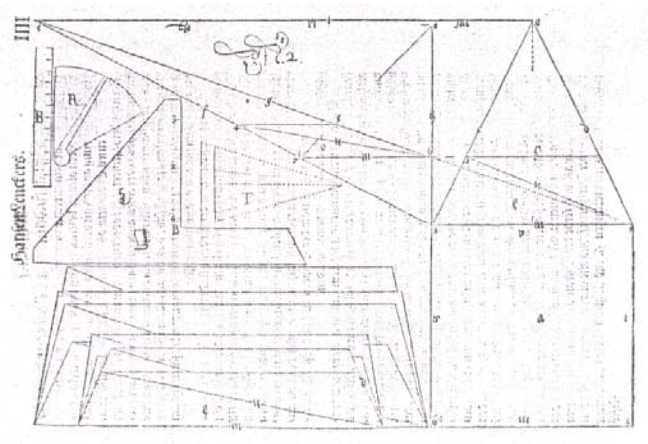
5. Johann II of Simmern: Perspective construction (fol. A5r)



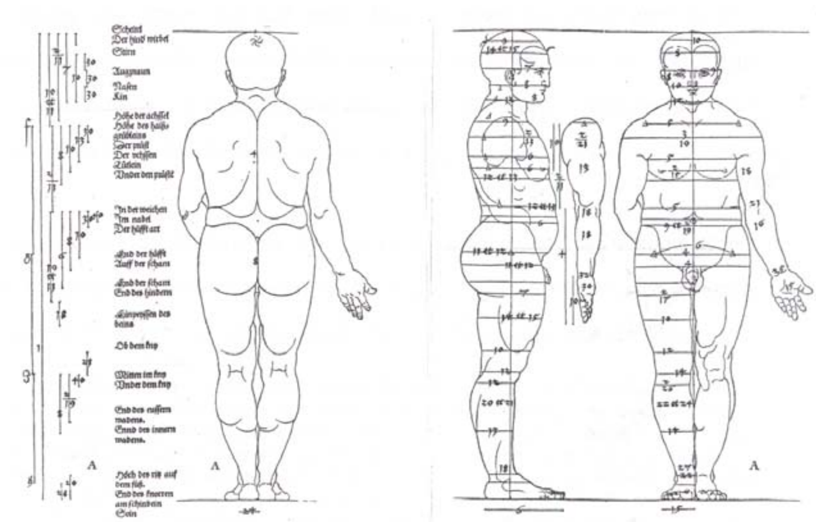
6. Heinrich Lautensack: Perspective construction (fol. 12r)



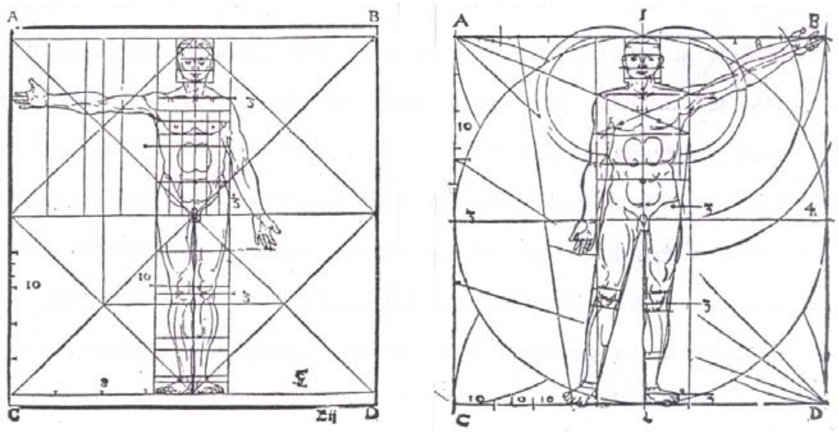
7. Augustin Hirschvogel: Perspective construction (p. C6r)



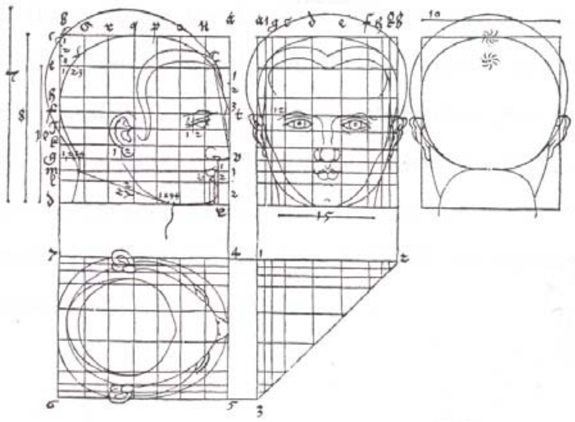
8. Hans Lencker: Perspective construction (pl. 2)



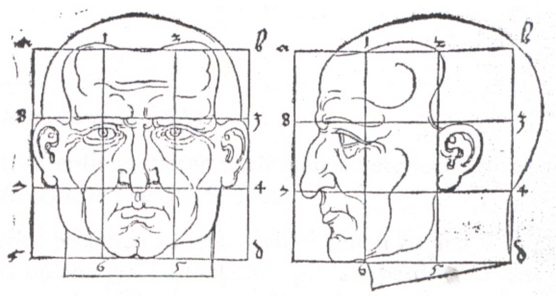
9. Albrecht Dürer: Constructing the human body (*Proportionslehre*, fol. A6r)



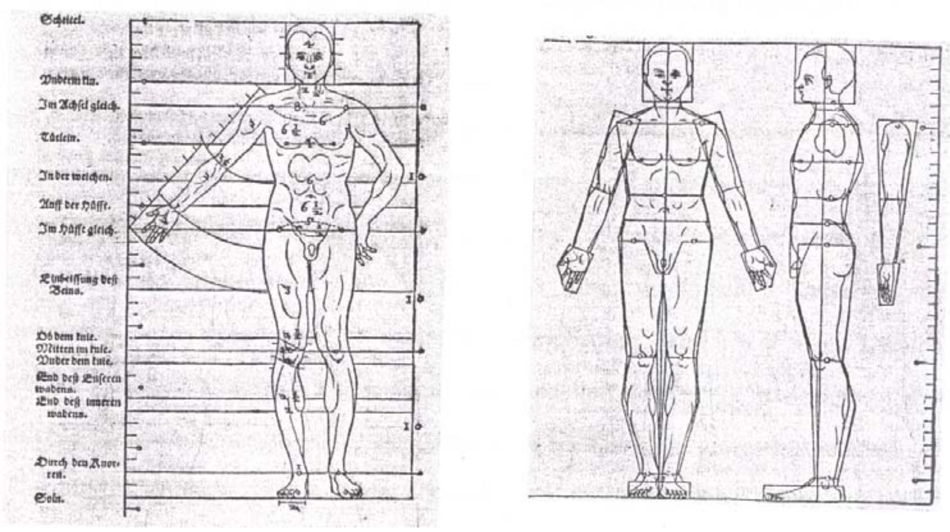
10. Erhard Schön: Proportional figure (pp. B2r and B4r)



11. Albrecht Dürer: Constructing the human face (*Proportionslehre*, fol. E2v)



12. Sebald Beham: Drawing the human face (p. B4r)



13. Heinrich Lautensack: Constructing the human body (fol. 34r and 36r)

Bibliography

Alberti, Leon Battista, *Das Standbild, Die Malkunst, Grundlagen der Malerei*, ed. O. Bätschmann and C. Schäublin, Darmstadt 2000

Arnulf, Arwed, 'Dürers Buchprojekte von 1511: Andachtsbücher für Humanisten', *Marburger Jahrbuch für Kunstwissenschaft*, XXXI, 2004, 145-74

Binding, Günther, 'Proportion', in *Lexikon des Mittelalters*, 10 vols, München 1970-99, VII, pp. 259-61

Bonnet, Anne-Marie, *'Akt' bei Dürer*, Köln 2001 (=Bonner Beiträge zur Renaissanceforschung, Bd. 4)

Cantor, Moritz, *Vorlesungen über Geschichte der Mathematik*, 4 vols, 2nd ed., Leipzig 1900

Clagett, Marshall, *Archimedes in the Middle Ages*, 5 vols, Philadelphia 1964-1984

Dürer, Albrecht, *Vnderweysung der messung / mit dem zirckel vnd richtscheyt / in Linien ebnen vnnd gantzen corporen*, Nuremberg 1525

Dürer, Albrecht, *Hierinn sind begriffen vier bücher von menschlicher Proportion / durch Albrechten Dürer von Nürenberg erfunden und be / schrieben / zů allen denen / so zů dieser kunst lieb tragen*, Nuremberg 1528

Folkerts, Menso, 'The Importance of the Latin Middle Ages for the Development of Mathematics', in M. F., *Essays on Early Medieval Mathematics. The Latin Tradition*, Aldershot 2003, pp. 1-24

Grote, Ludwig, 'Vom Handwerker zum Künstler. Das gesellschaftliche Ansehen Albrecht Dürers', in *Festschrift für Hans Liermann zum 70. Geburtstag*, ed. K. Obermayer and H.-R. Hagemann, Erlangen 1964, pp. 26-47

Homann, Frederick A., *Hugh of St Victor: Practical Geometry [Practica Geometriae]*, ed. and transl. by F. A. Homann, Milwaukee 1991

Kappel, Jutta, 'Elfenbeindrechselkunst am Dresdner Hof', in *In fürstlichem Glanz. Der Dresdner Hof um 1600*, ed. D. Syndram and A. Schermer, Hamburg, Dresden 2004, pp. 176-9

Keil, Robert, 'Zu Dürers frühen Proportionszeichnungen des menschlichen Körpers', *Pantheon*, XLIII, 1985, pp. 54-61

Kemp, Michael, *The Science of Art. Optical Themes in Western Art from Brunelleschi to Seurat*, New Haven 1992

Lencker, Hans, *Perspectiva literaria*, Nürnberg 1567

Lindgren, Uta, *Die Artes Liberales in Antike und Mittelalter. Bildungs- und wissenschaftsgeschichtliche Entwicklungslinien*, München 1992.

Olschki, *Geschichte der neusprachlichen wissenschaftlichen Literatur*, 3 vols, Heidelberg 1919, I, pp. 419-23.

Panofsky, Erwin, *The Life and Art of Albrecht Dürer*, Princeton 1971

Peiffer, Jeanne, 'Dürers Geometrie als Propädeutik zur Kunst', in *Wissenschaft-Technik-Kunst. Interpretationen-Strukturen-Wechselwirkungen*, ed. E. Knobloch, Wiesbaden 1997, pp. 89-103 (p. 91).

Plinius / König: C. Plinius Secundus d. Ä., *Naturkunde Lateinisch-Deutsch*, Buch XXXV: *Farben, Malerei, Plastik*, ed. and tr. (German) R. König, Darmstadt 1997, p. 64.

Price, David Hotchkiss, *Albrecht Dürer's Renaissance. Humanism, Reformation, and the Art of Faith*, Ann Arbor 2003

Rupprich, Hans, 'Die kunsttheoretischen Schriften L. B. Albertis und ihre Nachwirkung bei Dürer', *Schweizer Beiträge zur allgemeinen Geschichte*, XVIII/IXX, 1960/61, pp. 219-39

Schmid, Wolfgang, *Dürer als Unternehmer. Kunst, Humanismus und Ökonomie in Nürnberg um 1500*, Trier 2003

Schreiber, Peter, 'Albrecht Dürers Geometrie aus heutiger Sicht', in *Kepler-Symposium Philosophie und Geschichte der Mathematik. Vorträge 1995-2005*, ed. J. Maass, U. Langer and G. Lorcher, Linz 2005, pp. 85-107

Schwarz, Karl, *Augustin Hirschvogel. Ein deutscher Meister der Renaissance*, Berlin 1917
Scriba, Christoph J. and Peter Schreiber, *5000 Jahre Geometrie. Geschichte, Kulturen Menschen*, Berlin 2000
Shelby, Lon, 'The Geometrical Knowledge of the Mediaeval Master Mason', *Speculum*, XLVII, 1972, 395-421
Shelby, Lon, *Gothic Design Techniques. The Fifteenth-Century Design Booklets of Mathes Roriczer and Hanns Schmuttermayer*, Carbondale 1976 (referred to as Roriczer / Shelby)
Ullmann, B. L., 'Geometry in the Medieval Quadrivium', in *Studi di bibliografia e di storia in onore di Tammaro de Marinis*, 4 vols, Verona 1964, IV, pp. 263-85
Victor, Stephen K., *Practical Geometry in the High Middle Ages. 'Artis Cuiuslibet Consummatio' and the 'Praktike de Geometrie'*, Philadelphia 1979
Westfall, Carroll W., 'Painting and the Liberal Arts: Alberti's View', *Journal of the History of Ideas*, XXX, 1969, 487-506

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